DTP–MSU/00-14
DF/IST-5.2000
gr-qc/0008076

No-go theorem for false vacuum black holes

Dmitri V. Gal'tsov[1]
Department of Theoretical Physics,
Moscow State University, 119899, Moscow, Russia
and
José P. S. Lemos[2]
CENTRA, Departamento de Física, Instituto Superior Técnico,
Av. Rovisco Pais 1, 1096 Lisboa, Portugal

Abstract

We study the possibility of non-singular black hole solutions in the theory of general relativity coupled to a non-linear scalar field with a positive potential possessing two minima: a 'false vacuum' with positive energy and a 'true vacuum' with zero energy. Assuming that the scalar field starts at the false vacuum at the origin and comes to the true vacuum at spatial infinity, we prove a no-go theorem by extending a no-hair theorem to the black hole interior: no smooth solutions exist which interpolate between the local de Sitter solution near the origin and the asymptotic Schwarzschild solution through a regular event horizon or several horizons.

PACS numbers: 04.20.Jb, 04.50.+h, 46.70.Hg

[1]Supported by RFBR; Email: galtsov@grg.phys.msu.su
[2]E-mail: lemos@kelvin.ist.utl.pt

1 Introduction

In a recent paper Daghigh, Kapusta and Hosotani [1] proposed a new type of a black hole in the theory of general relativity coupled to a non-linear scalar field with a potential. A quartic non-symmetric potential was assumed to have one minimum of positive energy (false vacuum) and another of zero energy (true vacuum). In the false vacuum state, a possible solution is the de Sitter metric, while for the scalar field at the true vacuum one can assume the Schwarzschild metric. The 'false vacuum black hole' was suggested to be given by the Schwarzschild solution outside the event horizon and the de Sitter solution inside the black hole. The scalar field was supposed to have a constant false vacuum value inside the black hole and a constant true vacuum value outside with a finite jump at the horizon. Direct matching of de Sitter spacetime with the Schwarzschild solution on the horizon had been suggested earlier [2, 3] and shown to be incorrect [4, 5, 6]. However, it was argued in [1] that the matching of these solutions can be achieved within a more general parameterization of the static metric by two different functions due to the jump of the product $g_{tt}g_{rr}$. If correct, this proposal could lead to intriguing physical implications like new kinds of black hole remnants or the possibility that we live inside an enormous black hole.

Meanwhile, the problem of incorporating the de Sitter metric inside the black hole is by no means new. The idea that sigularitities can be avoided by matter with an 'inflationary' equation of state was forwarded by Gliner [7] long before the inflation scenario was proposed in cosmology. It was also suggested that the limiting curvature principle of Markov [8] may be realized in the black hole context as the appearence of a de Sitter world instead of the singularity [9, 10]. A similar issue was discussed using the idea of vacuum polarisation [5] and in the 'cutoff' curvature approach [11, 12]. Some phenomenological matter sources were suggested ensuring a de Sitter nucleus inside the black hole [13, 14]. It was also shown that, under perturbations, the de Sitter metric emerges inside a charged black hole in the development of an instability of the internal Cauchy horizon, a phenomenon called mass-inflation with an exponential growth of the local mass function [15]. Such a growth has also been observed inside static Einstein-Yang-Mills black holes when the singularity is approached [16, 17, 18]. A closely related subject – the avoidance of singularities inside black holes not neccessarily related to the de Sitter solution – has attracted much attention recently. Such a possibility was described by Bardeen in 1968 [19] as a modification of the Reissner-Nordström metric. Recently a non-linear electrodynamics lagrangian was found producing a Bardeen type metric [20]. Phenomenological sources for non-singular black holes were discussed in [21, 22]. In addition, we mention some investigations on the dynamics of time-dependent bubbles with false vacuum inside (and hence de Sitter metric inside) and a black hole metric outside (for a recent discussion see [23]). Thus, the main surprise of the paper [1] is a claim that an internal de Sitter metric can be accommodated to a black hole as a static solution within such a simple model as the scalar theory with a non-symmetric potential.

Here we investigate the problem of matching the de Sitter and the Schwarzschild metrics in the context of this model in more detail. First we show that the piecewise solution

suggested in [1] cannot be interpreted in terms of distributions and is likely to demand additional singular matter sources at the horizon. Then we discuss the possibility of more complicated smooth solutions assuming more general positive non-symmetric potential with two minima. First, using only a local analysis we explore the possibility of matching the internal and external solutions at the horizon attached to the local maximum of the potential. It turns out that both inside and outside the black hole one encounters timelike regions (with ∂_t spacelike). Finally, we discuss the problem from a global viewpoint, in the spirit of no-hair theorems. We prove a no-go theorem by extending the no-hair argument to the black hole interior and showing that the model does not admit smooth solutions in which the scalar field starts at the false vacuum at the origin and comes to the true vacuum at infinity with one or several horizons at finite values of the radii of spherical sections. Some open possibilities for false vacuum black holes are then briefly discussed.

2 The model

Consider general relativity coupled to a real scalar field theory

$$S = \frac{1}{4\pi} \int \left(-\frac{R}{4} + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right) \sqrt{-g}\, d^4x \tag{1}$$

with a smooth potential V for which we assume a general behavior as shown in Fig.1. In what follows the potential is not necessarily quartic. We need only that near minima, $-\phi_1$ and ϕ_2, it can be approximated by parabolae

$$\begin{aligned} V &= V_1 + \frac{1}{2} k_1 (\phi + \phi_1)^2 + O((\phi + \phi_1)^3), & (2) \\ V &= \frac{1}{2} k_2 (\phi - \phi_2)^2 + O((\phi - \phi_2)^3), & (3) \end{aligned}$$

with positive constants V_1, k_1, k_2, and the local maximum is at $\phi = 0$, with $V(0) = V_0 > 0$. We assume that the potential goes to infinity as a finite power of $|\phi|$ as $\phi \to \pm\infty$, and that the derivative

$$V_\phi = \frac{dV}{d\phi}. \tag{4}$$

is finite for finite ϕ.

Assuming spherical symmetry and staticity we write the metric in the curvature gauge

$$ds^2 = \sigma^2 N dt^2 - \frac{dr^2}{N} - r^2 d\Omega\,, \tag{5}$$

where σ and N are functions of r. For N we will also use the following two parametrizations:

$$N = 1 - \frac{2m}{r} = \frac{\Delta}{r}\,, \tag{6}$$

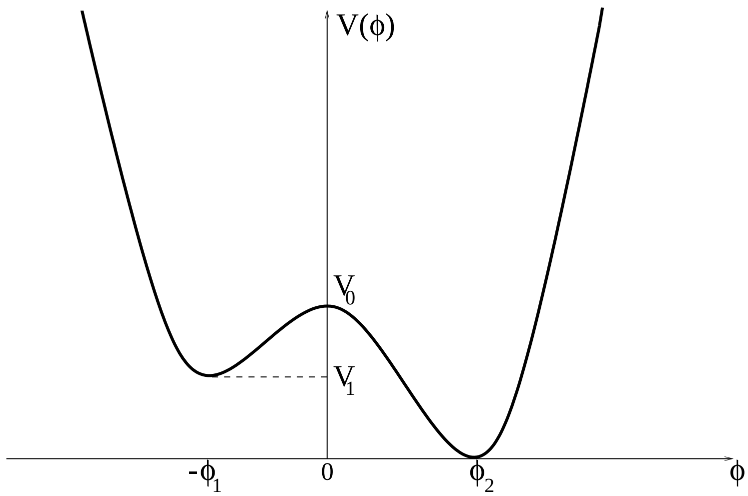

Figure 1: The potential V as a function of ϕ.

where $\Delta = r - 2m$. The equations of motion following from the action (1) read

$$\frac{\sigma'}{\sigma} = r\phi'^2, \tag{7}$$

$$m' = r^2\left(\frac{1}{2}N\phi'^2 + V\right), \tag{8}$$

$$\left(r^2 N\sigma\phi'\right)' = \sigma r^2 V_\phi. \tag{9}$$

The variable σ can be excluded from the Eq. (9) using Eq. (7). Denoting $\xi = r\phi'$, one can rewrite the equations of motion as a system of three first order equations for ϕ, ξ, Δ

$$\Delta' = -\frac{\Delta}{r}\xi^2 + U, \tag{10}$$

$$\Delta\xi' = r^2 V_\phi - \xi U, \tag{11}$$

$$\phi' = \frac{\xi}{r}, \tag{12}$$

where

$$U = 1 - 2r^2 V, \tag{13}$$

and an equation for σ, i.e. Eq. (7), which can be solved once the solution of the system (10-12) is found. A mechanical analogy is useful for the scalar field equation (9) regarding r as the mechanical time. Then the role of the potential will be played by $-V$ in the spacelike region ($\Delta > 0$) and by V in the timelike region ($\Delta < 0$, if any). Thus, a 'particle' departing from one of the vacua will be forced to climb the barrier if $\Delta > 0$, but will be turned back if $\Delta < 0$.

One trivial solution to the system (10-12) corresponds to the scalar field sitting at the

left local minimum of the potential (false vacuum):

$$\phi \equiv -\phi_1, \quad \sigma \equiv 1, \quad N = 1 - \frac{r^2}{r_c^2}. \tag{14}$$

This is the de Sitter spacetime with the cosmological horizon at

$$r = r_c = \sqrt{\frac{3}{2V_1}}. \tag{15}$$

Another trivial solution is the Schwarzschild metric. This corresponds to the true vacuum

$$\phi \equiv \phi_2, \quad \sigma \equiv 1, \quad N = 1 - \frac{M}{r}, \tag{16}$$

with finite mass M. Note that any constant value of the metric function σ may be rescaled to unity by a time scaling.

3 Piecewise solution

Let us try, following [1], to construct a piecewise solution to the Eqs. (10-12) assuming for the scalar field a step-like behaviour

$$\phi = -\phi_1\theta(r_h - r) + \phi_2\theta(r - r_h), \tag{17}$$

i.e., the false vacuum value inside the black hole and the true vacuum value outside. Similar matching within the parameterization of the metric by a single function of the radial variable was discussed earlier in [4, 6] and shown to lead to instabilities. Our current model differs in that the metric functions g_{tt} and g_{rr} are independent, so these results do not apply directly.

The field equations will be satisfied for $r < r_h$ with the de Sitter metric

$$\sigma = \sigma_0, \quad m = \frac{r^3}{2r_h^2}, \tag{18}$$

(where the constant value σ_0 can be removed by a rescaling of time), and for $r > r_h$ with the Schwarzschild metric

$$\sigma = \sigma_\infty, \quad m = \frac{r_h}{2}, \tag{19}$$

the event horizon coinciding with the de Sitter cosmological horizon. This suggests the following representation for the metric functions:

$$\sigma = \sigma_0\theta(r_h - r) + \sigma_\infty\theta(r - r_h), \tag{20}$$

$$m = \frac{r^3}{2r_h^2}\theta(r_h - r) + \frac{r_h}{2}\theta(r - r_h). \tag{21}$$

However, an attempt to check whether this solution remains true at the horizon $r = r_h$ in the sense of distributions fails because the system (10-12) is highly non-linear. This could be expected: distribution-valued metrics and curvature can be introduced in general relativity only under special conditions (see, e.g., [24]). Perhaps more satisfactory results could be obtained in the framework of the generalized functions approach by Colombeau [25], but it will be enough for our purposes here to use a simple regularization for the delta-function

$$\delta(x) = \lim_{\varepsilon \to 0} \delta_\varepsilon(x), \quad \delta_\varepsilon(x) = \frac{\varepsilon}{\pi(x^2 + \varepsilon^2)}. \tag{22}$$

Differentiation of Eq. (17) gives

$$\xi = r_h(\phi_1 + \phi_2)\delta(x), \quad x = r - r_h, \tag{23}$$

while for the derivative of m we obtain

$$m' = \frac{3(1+x)^2}{2}\theta(-x). \tag{24}$$

Near $x = 0$ one also has

$$\Delta = x\left[\theta(x) - 2\theta(-x)\right], \tag{25}$$

therefore at the right hand side of the Eq.(8) one encounters a product $x\delta^2(x)$. Its regularized version

$$x\delta_\varepsilon(x)^2 = \frac{\varepsilon^2}{\pi^2}\frac{x}{(x^2 + \varepsilon^2)^2} \tag{26}$$

vanishes at $x = 0$. Moreover, it can be checked that for any 'good' function $\varphi(x)$ one has:

$$\lim_{\varepsilon \to 0} \int \varphi(x) x \delta_\varepsilon^2(x) dx = 0. \tag{27}$$

Dropping therefore the first term in the right hand side of Eq. (8) one can verify the validity of this equation at $x = 0$ in view of the formula

$$V = \frac{3}{2r_h^2}\theta(-x). \tag{28}$$

However, the situation with other equations is less satisfactory. For the solution under consideration $V_\phi = 0$, while the left hand side of the Eq. (9) contains a delta-singularity. Then Eq. (7) shows that the jump of σ at $x = 0$ is infinite:

$$\ln \sigma \Big|_{r_h - 0}^{r_h + 0} = r_h(\phi_1 + \phi_2)^2 \delta(0), \tag{29}$$

(this is easy to check using the regularization (22)). The structure of other singularities in the Einstein tensor is likely to exhibit the presence of extra matter at the horizon surface.

The Einstein tensor for the metric (5) reads:

$$
\begin{aligned}
G_t^t &= \frac{2m'}{r^2}, && (30)\\
G_r^r &= \frac{2}{r^2}\left(m' - \Delta\frac{\sigma'}{\sigma}\right), && (31)\\
G_\theta^\theta &= -\frac{1}{2r}\left(\Delta'' + 2\Delta\frac{\sigma''}{\sigma} + \left(3\Delta' - \frac{\Delta}{r}\right)\frac{\sigma'}{\sigma}\right), && (32)
\end{aligned}
$$

while the right hand side of the Einstein's equations for the scalar field is given by

$$
\begin{aligned}
8\pi T_t^t &= N\phi'^2 + 2V, && (33)\\
8\pi T_r^r &= -N\phi'^2 + 2V, && (34)\\
8\pi T_\theta^\theta &= N\phi'^2 + 2V. && (35)
\end{aligned}
$$

The tt-equation coincides with the Eq.(8) and it is satisfied at the horizon. The rr-equation is satisfied as well after omitting the products $x\delta^2(x)$ as we argued above. But the situation is more complicated with the $\theta\theta$-equation. Indeed, differentiating (25) one obtains a delta-term

$$\Delta'' = 3\delta(r - r_h), \tag{36}$$

which is not canceled by other terms in this equation. The same is true for the singular term containing $\Delta'\sigma'/\sigma$. All this looks like showing the presence of extra matter at the horizon. This is not very surprising: the piecewise model emerges when the scalar theory is treated like a vacuum theory with a step-like cosmological constant. But if the cosmological constaint is variable, the Bianchi identities require additional matter to be invoked [26].

Therefore it is hard to adopt the piecewise metric (20), (21) as a true solution to the Einstein equations with a scalar source.

4 An attempt of a smooth matching at $\phi = 0$

One can imagine that a true solution with similar properies exists which is deformed from the above simple form in the vicinity of the horizon. Here we perform a purely local analysis of the behavior of the presumed smooth solution near the horizon assuming that the latter corresponds to a 'natural' point $\phi = 0$, i.e., to the local maximum of the potential shown in Fig. 1. Then $V_\phi(r_h) = 0$ and we get from the Eqs. (10,11)

$$\Delta'_h = U_h, \quad \xi_h U_h = 0. \tag{37}$$

(Here we also used the fact that $\Delta\xi^2$ vanishes at the horizon, this remains true even if ξ diverges, see the next section). So either

$$i) \quad U_h = 0, \quad \xi_h \text{ arbitrary},$$

or

$$ii) \quad \xi_h = 0, \quad U_h \text{ arbitrary.}$$

Consider first the case i). Then $\Delta'_h = 0$ and the horizon is degenerate (Δ has a zero of the second order). Therefore near the horizon

$$\Delta = \frac{\alpha}{2} x^2 + O(x^3), \tag{38}$$

where $x = r - r_h$, and α should be positive to ensure the timelike character of the Killing vector ∂_t outside the horizon. Now expand the Eq. (10) to linear order in x. Equating the linear terms one gets:

$$\alpha = U'_h, \tag{39}$$

with U'_h being the value of the derivative of (13) at the horizon. It can be represented as

$$U'_h = -4 r_h V_h - 2 r_h^2 V_\phi \Big|_{r_h} \phi'_h, \tag{40}$$

where, by the assumption that the horizon is at the maximum of the potential,

$$V_\phi \Big|_{r_h} = 0, \tag{41}$$

and by the assumption i)

$$2 r_h^2 V_h = 1. \tag{42}$$

It follows that

$$\alpha = -\frac{2}{r_h}, \tag{43}$$

therefore ∂_t is spacelike outside the horizon. By definition, the event horizon is the largest root of the equation $\Delta = 0$, so this solution can not describe a black hole.

Consider now the case ii). If $\xi_h = 0$ and $U_h \neq 0$ an expansion for Δ will contain a linear term

$$\Delta = U_h x + \frac{\alpha}{2} x^2 + O(x^3), \tag{44}$$

while ξ starts as

$$\xi = \xi'_h x + O(x^2). \tag{45}$$

Now, collecting linear terms in the Eq. (10) we get again Eq. (43) for α, so the expansion of Δ will read

$$\Delta = -\frac{1}{r_h} \left(x - \frac{1}{2} U_h r_h \right)^2 + \frac{1}{4} U_h^2 r_h + O(x^3)\,. \tag{46}$$

Now the region with the timelike ∂_t is between the horizons, while outside it ∂_t is spacelike again.

So the result of our attempt is disapointing: one cannot smoothly match the solutions with both interior and exterior spacelike metrics at the event horizon attached to the 'natural' point $\phi = 0$.

It is worth noting that the matching of de Sitter and Reissner-Nordström metrics *is* possible along the interior Cauchy horizon for some special choice of parameters [27].

5 No-go theorem

Now we would like to study the problem in a more general setting: whether a static spherically symmetric smooth solution to the model (1) exists which is locally de Sitter near the origin (with ∂_t timelike) and asymptotically Schwarzschild with one or several regular horizons in the intermediate region. More precisely, we will assume that near the origin the space-time is flat:

$$N(0) = 1, \quad \sigma(0) = \sigma_0 \neq 0, \tag{47}$$

(the constant value σ_0 can be eliminated locally by rescaling of time), and the scalar field is at the false vacuum

$$\phi(0) = -\phi_1; \tag{48}$$

while at infinity the solution is asymptotically flat

$$N = 1 - \frac{2M}{r} + O\left(\frac{1}{r^2}\right), \quad \sigma(\infty) = 1, \tag{49}$$

and the scalar field is at the true vacuum:

$$\phi(\infty) = \phi_2. \tag{50}$$

We will give a non-existence proof in two steps: first we invoke the no-hair argument for the exterior region, and then we extend it to the black hole interior. The exterior no-hair theorem for an Abelian Higgs model was proven by Adler and Pearson [28] (for an improved version see [29, 30]). In [28] the Goldstone model was also considered for the Mexican hat potential. Here we deal with the real scalar field with a more general positive potential, so it is worth giving the no-hair proof explicitly.

The finiteness of the ADM mass M imposes restrictions on the first subleading term in the asymptotic expansion of the scalar field at infinity. Integrating the Eq. (8) from the event horizon (the maximal root of Δ) to infinity we find for the ADM mass

$$M = \frac{r_h}{2} + \int_{r_h}^{\infty} \left(\frac{1}{2}N\xi^2 + r^2 V\right) dr, \tag{51}$$

where the first term is the 'bare' mass, and the second one is the contribution of the scalar hair. Both terms in the integrand are positive semidefinite, so each of them should be integrable. For convergence of the second term at infinity it is necessary that V as a function of r decays faster than r^{-3}. In view of the Eq. (3) this translates to

$$\phi = \phi_2 + o\left(\frac{1}{r^{3/2}}\right), \tag{52}$$

as $r \to \infty$. This implies

$$\xi = o\left(\frac{1}{r^{3/2}}\right), \tag{53}$$

so the first term in Eq. (51) is also integrable at infinity. From the Eq. (7) it then follows that at infinity

$$\sigma = 1 - o\left(\frac{1}{r^3}\right), \tag{54}$$

with a negative subleading term, where the constant was set to unity in conformity with the assumption (49). With this normalization, an integration of the Eq. (7) gives

$$\sigma = \exp\left(-\int_r^\infty \xi^2 \frac{dr}{r}\right), \tag{55}$$

so σ is bounded to the interval

$$0 < \sigma \le 1. \tag{56}$$

Here the lower bound $\sigma = 0$ could be reached at a point where the integral in the exponential diverges. However in view of the assumption (47) σ remains non-zero at the origin, and being a non-decreasing function, remains strictly positive elsewhere.

We now discuss the behaviour of the solution near the event horizon. An assumption of regularity implies that the mixed components of the energy-momentum tensor (33-35) are finite, so ξ should satisfy

$$\lim_{r\to r_h} N\xi^2 = C^2, \tag{57}$$

with some $C \ge 0$, while ϕ should not diverge in view of our assumptions about the potential V (also implying the finiteness of the derivative V_ϕ). These conditions ensure the convergence of the integral (51) at the horizon. One can obtain a stronger condition on C using the field equations. From the Eqs. (10,11) one derives

$$(rN\xi)' + \frac{N}{r}\xi^3 = r^2 V_\phi. \tag{58}$$

Substituting here ξ from the Eq. (57) we obtain in the right vicinity of the horizon

$$\frac{CN'r_h}{2\sqrt{N}} + \frac{C^3}{\sqrt{N}} + \text{finite terms} = r^2 V_\phi. \tag{59}$$

The right-hand side of this equation remains finite as $r \to r_h$, while the first two terms on the left-hand side are positive semidefinite (recall that $N'_h \ge 0$) and diverge (for a degenerate horizon only the second term diverges). Therefore $C = 0$, i.e. we get a stronger condition on ξ:

$$\lim_{r\to r_h} N\xi^2 = 0. \tag{60}$$

(An alternative proof of this relation follows from the convergence of the integral in the exponential in the Eq. (55).) From the Eq. (57), a weaker condition also holds

$$\lim_{r\to r_h} N\xi = 0. \tag{61}$$

Now we can give a no-hair proof similar to that of Adler and Pearson [28].From the Eqs. (9), (10) and (7) one derives the following identity:

$$\frac{d}{dr}\left[\sigma\left(2r^2V-\frac{\Delta\xi^2}{r}\right)\right]=\sigma\left[\frac{\xi^2}{r}\left(1+\frac{\Delta}{r}\right)+4rV\right]. \tag{62}$$

Integrating it from the horizon to infinity and taking into account the condition (60), boundedness of σ, and the relations valid at $r\to\infty$

$$r^2V = o\left(\frac{1}{r}\right), \tag{63}$$

$$\frac{\xi^2\Delta}{r} = o\left(\frac{1}{r^3}\right), \tag{64}$$

which follow from the Eqs. (3), (52) and (53), we obtain

$$\int_{r_h}^{\infty}\sigma\left[\frac{\xi^2}{r}\left(1+\frac{\Delta}{r}\right)+4rV\right]dr=-2r_h^2V_h\sigma_h, \tag{65}$$

where V_h is the value of the potential at the event horizon. Since σ is strictly positive, the potential is positive semidefinite, and $\Delta\geq 0$ everywhere in the integration region, it follows that both the left hand side and the right hand side of this equation are strictly zero, which implies

$$\phi\equiv\phi_2 \tag{66}$$

for all $r\geq r_h$. Thus the event horizon should correspond to the absolute minimum (true vacuum) of the potential. Consequently, the black hole is exactly Schwarzschildean for an external observer.

Now let us look to the interior of the black hole. Since we assume that the Killing vector ∂_t is timelike in the vicinity of the origin, there are two possibilities: either there exist an internal Cauchy horizon at some $r=r_-$ (more generally an odd number of internal horizons), or the event horizon at $r=r_h$ is degenerate. The second possibility is a limiting case of the first, so we assume that there are two solutions $r=r_\pm$ of the equation $\Delta=0$ such that $r_-\leq r_+=r_h$ (the generalization of the following proof to a finite odd number of internal horizons is straightforward). Generalizing the condition (61) we find

$$\lim_{r\to r_\pm}N\phi'=0. \tag{67}$$

At the origin ϕ has a finite limit $-\phi_1$ and is (by assumption) a smooth function, therefore

$$\lim_{r\to 0}r^2\phi'=0. \tag{68}$$

Then, integrating the Eq. (9) form the origin to r_-, and taking into account boundedness of σ we get the relation

$$r^2\sigma N\phi'\Big|_0^{r_-}=0=\int_0^{r_-}\sigma r^2V_\phi dr. \tag{69}$$

It follows that either

$$i) \quad \phi(r) = -\phi_1 \quad \text{for all } \; r \in [0, r_-],$$

i.e. the solution is exactly de Sitter up to $r = r_-$ in which case $r_- = r_c$ given by the Eq. (15), or

$$ii) \quad \phi(r_-) \in (0, \phi_2],$$

i.e. the inner horizon is attached to the right wing $(0, \phi_2]$ of the potential curve, see Fig.1.

The case i) means that there is a global solution such that ϕ is identically constant in the finite interval $[0, r_-]$, ϕ is equal to a different constant along the semi-axis $[r_+, \infty)$, but ϕ is varying in the interval (r_-, r_+). Impossibility of a smooth solution of such a kind can be made clear from a mechanical analogy: the particle sitting at the bottom of the potential well for a finite time cannot start moving unless it is pushed, since all the derivatives of ϕ at the initial moment are zero by continuity. To get an idea how this follows from the field equations, consider a vicinity of the event horizon $r = r_+$, where $V = 0$ and hence $U = 1$, while $\Delta = r - r_+ = x$. Then the leading terms in the Eq. (11) give the following equation for ξ:

$$x\xi' = -\xi. \tag{70}$$

The singular solution $\xi = 1/x$ for $x < 0$ can not be matched to $\xi \equiv 0$ for $x > 0$ (and does not satisfy (60)), therefore the correct solution is $\xi = 0$.

Now consider the case ii). Integrating the Eq. (9) from r_- to r_+ we obtain

$$r^2 \sigma N \phi' \Big|_{r_-}^{r_+} = 0 = \int_{r_-}^{r_+} \sigma r^2 V_\phi dr. \tag{71}$$

Here $V_\phi \leq 0$, and therefore $V_\phi \equiv 0$ in $[r_-, r_+]$ which implies

$$r_- = r_+, \quad \phi(r_-) = \phi_2, \tag{72}$$

that is the event horizon is degenerate. But then we come back to the step like solution (17) which faces the problems discussed in the Sec. 2 and should be ruled out by the smoothness assumption. This completes the proof.

6 Conclusion

Our results are the following. First, we have shown that the piecewise false vacuum black hole presented in [1] can not be interpreted in terms of distributions and apparently is not a solution to the Einstein-scalar field equations without additional matter sources. Second, we have extended the no-hair argument to the black hole interior and have shown that there are no smooth solutions to the scalar model with a non-symmetric potential which interpolate from the false vacuum inside (in the region with the timelike Killing vector ∂_t) and the true vacuum outside through the horizon(s).

One could ask whether it is possible to weaken some of the assumptions made in order to reopen the possibility of such or similar configurations. The first assumption is the positivity and the shape of the potential. In fact, for static spherically symmetric configurations one can invert the roles of the scalar field ϕ and the potential $V(\phi)$: one chooses the desired behavior of ϕ determining afterwards the potential through the equations. In such a way some potentials were found for which scalar hair does exist [31, 32]. The main problem here is, of course, whether these potentials are physically reasonable. The second assumption is asymptotic flatness. It was found [33] that in an asymptotically de Sitter spacetime scalar hair does exist within a similar model. Finally one could consider a non-minimal coupling where no-hair theorems also get modified [34, 35].

Acknowledgements

The authors thank anonymous referees for indicating the references [2, 3, 4, 6, 27] and useful comments. D.G. is grateful to CENTRA/Instituto Superioro Técnico (Lisbon) and GTAE for hospitality and support in August 2000 when this paper was written. His work was also partially supported by the Russian Foundation for Basic Research under the grant 00-02-16306. JPSL acknowledges a grant from FCT through the project ESO/PRO/1250/98, and thanks Observatório Nacional - Rio de Janeiro for hospitality.

References

[1] R. G. Daghigh, J. I. Kapusta and Y. Hosotani, False vacuum black holes and Universes, 2000, gr-qc/0008006.

[2] P. F. Gonzales-Diaz, Lett. Nuovo Cim. **32** (1981) 161.

[3] W. Shen and S. Zhu, Phys. Lett. **A126** (1988) 229.

[4] O. Gron, Lett. Nuovo Cim. **44** (1985) 177.

[5] E. Poisson and W. Israel, Class. Quant. Grav. **5** (1988) L201.

[6] O. Gron and H. Soleng, Phys. Lett. **A138** (1989) 89.

[7] E. Gliner, Sov. Phys. JETP, **22** (1966) 378; see also gr-qc/9808042.

[8] M. A. Markov, Ann. Phys. (NY) **155** (1984) 333.

[9] V. P. Frolov, M. A. Markov, and V. F. Mukhanov, Phys. Lett. **B216** (1989) 272; Phys. Rev. **D41** (1990) 3831.

[10] C. Barrabes and V. P. Frolov, Phys. Rev. **D53** (1996) 3215.

[11] J. Polchinski, Nucl. Phys. **B 325** (1989) 619.

[12] D. Morgan, Phys. Rev. **D43** (1991) 3144.

[13] I. G. Dymnikova, Gen. Rel. Grav. **24** (1992) 235.

[14] I. G. Dymnikova, Phys. Lett. **B472** (2000) 33.

[15] E. Poisson and W. Israel, Phys. Rev. **D41** (1990) 1796.

[16] E.E. Donets, D.V. Gal'tsov, and M.Yu. Zotov, Phys. Rev. **D 56** (1997) 3459.

[17] D.V. Gal'tsov, E.E. Donets, and M.Yu. Zotov, JETP Lett. **65** (1997) 895.

[18] P. Breitenlohner, G. Lavrelashvili, and D. Maison, Nucl. Phys. **B 524** (1998) 427.

[19] J. Bardeen, in Proceedings of GR5, Tbilisi, URSS, 1968.

[20] E. Ayón-Beato and A. Garcia, Phys. Rev. Lett. **80** (1998) 5056.

[21] M. Mars, M. M. Martin-Prats and J. M. M. Senovilla, Class. Quant. Grav. **13** (1996) L51.

[22] A. Borde, Phys. Rev. **D55** (1997) 7615.

[23] G. L. Alberghi, D. A. Lowe and M. Trodden, JHEP **9907** (1999) 020.

[24] D. Garfinkle, Class. Quant. Grav. **16** (1999) 4101.

[25] J. F. Colombeau, *New Generalized Functions and Multiplication of Distributions (North-Holland Mathematics Studies 84)*, Amsterdam, North-Holland, 1984.

[26] J. M. Overduin and F. I. Cooperstock, Phys. Rev. **D58** (1998) 043506.

[27] C. Barrabes and W. Israel, Phys. Rev. **D43** (1991) 1129.

[28] S. L. Adler and R. B. Pearson, Phys. Rev. **D18** (1978) 2798.

[29] G. W. Gibbons, Lecture Notes in Phys., **383**, 110-133. Springer-Verlag, Berlin, 1991.

[30] A. Lahiri, Mod. Phys. Lett. **A8** (1993) 1549.

[31] O. Bechmann and O. Lechtenfeld, Class. Quant. Grav. **12** (1995) 1473.

[32] H. Dennhardt and O. Lechtenfeld, Int. J. Mod. Phys. **A13** (1998) 741-764.

[33] T. Torii, K. Maeda and M. Narita, Phys. Rev. **D59** (1999) 064027.

[34] A. E. Mayo and J. D. Bekenstein, Phys. Rev. **D54** (1996) 5059.

[35] J. D. Bekenstein, in: Proc. of the Second Sakharov Conference in Physics, Moscow, 1966. Eds. I. M. Dremin and A. M. Semihatov, World Scientific, Singapore, 1997. gr-qc/9605059.